UNIVERSITY OF KARLSRUHE
Institute for Hydromechanics

Mixing, Transport, and Transformation

Lecture 9:
Mixing in Lakes and Reservoirs

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Mixing in Lakes and Reservoirs

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1. Concepts and definitions

Mixing in inland lakes and reservoirs belongs to the field of physical limnology (the study of the physical properties of bodies of fresh water).

In general, lakes and reservoirs are stratified systems. Nearly every water body is stratified to some degree, but the stratification in lakes and reservoirs is strong enough to play a dominant role in mixing.

Lakes and reservoirs are stratified for three main reasons:

1. Lakes and reservoirs are comparatively stagnant, having low flow velocities, often laminar in nature.
2. Lakes and reservoirs have long residence times.
3. Lakes and reservoirs form in (sometimes deep) depressions which naturally reduce the interaction between surface and bottom water.
2. Energy budget

From a fundamental view point the total energy in a lake is the sum of the thermal and mechanical energy:

\[ \frac{dE_{\text{tot}}}{dt} = \frac{dE_P}{dt} + \frac{dE_K}{dt} + \frac{dE_T}{dt} \] (1)

Figure 1 shows a schematic of all the major energy inputs to a typical lake. These inputs can be categorized as

- Radiation
- Evaporation and condensation
- Wind
- Direct inflows and outflows
3. Seasonal lake cycle

The seasonal cycle of lake stratification results from the seasonal variation in solar radiation and controls the long-term temperature and distribution of chemicals (in particular oxygen) in the lake. Figure 2 shows a schematic of this seasonal cycle for a temperate lake.

The three main lake states are

1. Summer stratification.
2. Fall and spring turnover.
3. Winter inverse stratification

Lakes that experience this full cycle of stratification states are called dimictic because they stratify twice. Lakes that only stratify in the summer are called monomictic.
4. Transport and mixing mechanisms

In general, the active mixing processes in a lake or reservoir are driven by diurnal (daily) forcing. Figure 3, taken directly from Imboden & Wüest (1995), illustrates the common types of mixing mechanisms in lakes and reservoirs. These are due mainly to the action of:

- Wind.
- Boundary mixing.
- Inflows and outflows.
- Radiation.
- Chemistry.
5. Lake mixing regimes

As is clear from the previous section, many processes are responsible for mixing in lakes and reservoirs. A non-dimensional number that expresses the mixing potential of a shear flow in a stably stratified ambient is the Richardson number, $Ri$, defined as

$$Ri = -\frac{g}{\rho_0} \frac{\partial \rho}{\partial z} \left( \frac{\partial u}{\partial z} \right)^2.$$  \hspace{1cm} (2)

Other similar numbers can be developed to describe the mixing potential of other mixing mechanisms.

For a strongly stratified reservoir acted on by wind mixing, we can use a modified version of $Ri$ to classify the mixing regime. The velocity gradient in the surface water is proportional to the shear velocity due to the wind, given by

$$u_s = \left( \frac{\rho_a}{\rho_w} \frac{U_{20}^2}{10} \right)^{1/2}$$  \hspace{1cm} (3)
where \( \rho_a \) is the density of the air, \( \rho_w \) is the density of the water and \( U_{10} \) is the wind speed at an elevation of 10 m above the water. The term \((g \Delta \rho) / \rho\) can be approximated as the reduced gravity between the epilimnion and the hypolimnion, namely

\[
g' = g \frac{\rho_h - \rho_e}{\bar{\rho}}
\]  \hspace{1cm} (4)

where \( \rho_h \) is the hypolimnion density, \( \rho_e \) is the epilimnion density, and \( \bar{\rho} \) is the average lake density. Finally, the density differences occur over a depth about equal to the depth of mean density, or

\[
h = z(\bar{\rho})
\]  \hspace{1cm} (5)

where \( h \) is the depth at which the density is equal to \( \bar{\rho} \). Combining these approximations, the Richardson number is

\[
R_i = \frac{g' n}{u^2}
\]  \hspace{1cm} (6)
This form of $Ri$ gives the ratio of the stability due to the stratification compared to the instability caused by the wind stirring.

Fischer et al. (1979) suggest four mixing regimes for different ranges of $Ri_*$. The values of $Ri_*$ are compared to the lake mean depth, $h$, and the length of the wind fetch across the lake $L$. For the derivation of these critical regimes refer to Fischer et al. (1979). The regimes are defined in Fischer et al. (1979) as follows:

1. Regime A: $Ri_* > L^2/(2h)^2$. The deepening process proceeds very slowly by turbulent erosion.
2. Regime B: $L/(2h) < Ri_* < L^2/(2h)^2$. Internal waves are the predominant feature of this regime.
3. Regime C: $1 < Ri_* < L/(2h)$. Throughout this regime the thermocline will be diffuse and steeply inclined.
4. Regime D: $Ri_* > 1$. Deepening is now so rapid and chaotic that the interface will not be well defined.
These mixing regimes and their development as the wind persists over time are illustrated in Figure 4.

Similar exercises and modified $Ri$ numbers can be used to evaluate the strength of mixing for the other mechanisms described in the previous section. Interested readers are pointed to a series of lectures by Jörg Imberger available over the web\(^1\).

\(^1\) [http://www.cwr.uwa.edu.au/Presentations/](http://www.cwr.uwa.edu.au/Presentations/)
Figure 1: Schematic of the energy inputs to a lake.
Figure 2: Schematic of the seasonal cycle in a dimictic lake.
Figure 3: Schematic of the mixing processes in a lake.
Figure 4: Schematic of the mixing sequences for the different regimes of mixed layer deepening taken directly from Fischer et al. (1979).
References
