

Environmental Fluid Mechanics

Part I: Mass Transfer and Diffusion

Engineering – Lectures

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Environmental Fluid Mechanics Part I: Mass Transfer and Diffusion

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Course Syllabus

Lecture	Chapter	Type	Content	Exercises
21.10.02 11.30–13.00	—	V1	Introduction. Course outline, introduction and examples of transport problems.	
28.10.02 11.30–13.00	Ch. 1	V2	Fick's Law and the Diffusion Equation. Derivation of the diffusion equation using Fick's law.	
28.10.02 15.45–17.15	Ch. 1	V3	Point Source Solution. Similarity method solution and comparison with Gaussian distribution.	HW1 out
04.11.02 11.30–13.00	Ch. 2	V4	Advective-Diffusion Equation. Derivation of the advective-diffusion (AD) equation using coordinate transformation.	
04.11.02 15.45–17.15	Ch. 2	Ü1	Diffusion. Solving diffusion problems using known solutions and superposition.	
11.11.02 11.30–13.00	Ch. 3	V5	Turbulence. Introduction to turbulence and the mathematical description of turbulence.	HW1 in
11.11.02 15.45–17.15	Ch. 3	V6	Turbulent Diffusion. Reynold's averaging, the turbulent AD equation, and turbulent mixing coefficients.	HW2 out
18.11.02 11.30–13.00	Ch. 3	V7	Longitudinal Dispersion. Taylor dispersion and derivation of the dispersion coefficient.	
18.11.02 15.45–17.15	Ch. 3	Ü2	Dispersion. Taylor dispersion in a pipe.	
25.11.02 11.30–13.00	Ch. 4	V8	Chemical, Physical and Biological Transformation. Transformation and its incorporation in the AD equation.	HW2 in
25.11.02 15.45–17.15	Ch. 5	V9	Mixing at the Air-Water Interface. Exchange at the air-water interface and aeration models.	HW3 out
02.12.02 11.30–13.00	Ch. 5	V10	Mixing at the Sediment-Water Interface. Exchange at the sediment-water interface.	
02.12.02 15.45–17.15	Ch. 6	V11	Atmospheric Mixing. Turbulence in the atmospheric boundary layer and transport models.	
09.12.02 11.30–13.00	Ch. 7	V12	Water Quality Modeling. Water quality modeling methodology and introduction to simple transport models.	HW3 in
09.12.02 15.45–17.15	All	Ü3	Review. Course review with sample exam problems.	HW4 out

Recommended Reading

Journal Articles

Journals are a major source of information on Environmental Fluid Mechanics. Three major journals are the *Journal of Fluid Mechanics* published by Cambridge University Press, the *Journal of Hydraulic Engineering* published by the American Society of Civil Engineers (ASCE) and the *Journal of Hydraulic Research* published by the International Association of Hydraulic Engineering and Research (IAHR).

Supplemental Textbooks

The material for this course is also treated in a number of excellent books; in particular, the following supplementary texts are recommended:

Acheson, D. J. (1990), *Elementary Fluid Dynamics*, Oxford Applied Mathematics and Computing Science Series, Clarendon Press, Oxford, England.

Fischer, H. B., List, E. G., Koh, R. C. Y., Imberger, J. & Brooks, N. H. (1979), *Mixing in Inland and Coastal Waters*, Academic Press, New York, NY.

Mei, C. C. (1997), *Mathematical Analysis in Engineering*, Cambridge University Press, Cambridge, England.

Condensed Bibliography

Csanady, G. T. (1973), *Turbulent Diffusion in the Environment*, D. Reidel Publishing Company, Dordrecht, Holland.

Kundu, P. K. & Cohen, I. M. (2002), *Fluid Mechanics*, 2nd Edition, Academic Press, San Diego, CA.

Rutherford, J. C. (1994), *River Mixing*, John Wiley & Sons, Chichester, England.

van Dyke, M. (1982), *An Album of Fluid Motion*, The Parabolic Press, Stanford, California.

Wetzel, R. G. (1983), *Limnology*, Saunders Press, Philadelphia, PA.

VIII Recommended Reading

Preface

Environmental Fluid Mechanics (EFM) is the study of motions and transport processes in earth's hydrosphere and atmosphere on a local or regional scale (up to 100 km). At larger scales, the Coriolis force due to earth's rotation must be considered, and this is the topic of Geophysical Fluid Dynamics. Sticking purely to EFM in this book, we will be concerned with the interaction of flow, mass and heat with man-made facilities and with the local environment.

This text is organized in two parts and is designed to accompany a series of lectures in a two-semester course in Environmental Fluid Mechanics. The first part, Mass Transfer and Diffusion, treats passive diffusion by introducing the transport equation and its application in a range of unstratified water bodies. The second part, Stratified Flow and Buoyant Mixing, covers the dynamics of stratified fluids and transport under active diffusion.

The text is designed to compliment existing text books in water and air quality and in transport. Most of the mathematics are written out in enough detail that all the equations should be derivable (and checkable!) by the reader. This second edition adds several example problems to each chapter and expands the homework problem sections at the end of each chapter. Solutions to odd-numbered homework problems have also been added to Appendix ??.

This book was compiled from several sources. In particular, the lecture notes developed by Gerhard H. Jirka for courses offered at Cornell University and the University of Karlsruhe, lecture notes developed by Scott A. Socolofsky for courses taught at the University of Karlsruhe, and notes taken by Scott A. Socolofsky in various fluid mechanics courses offered at the Massachusetts Institute of Technology (MIT), the University of Colorado, and the University of Stuttgart, including courses taught by Heidi Nepf, Chiang C. Mei, Eric Adams, Ole Madsen, Ain Sonin, Harihar Rajaram, Joe Ryan, and Helmut Kobus. Many thanks goes to these mentors who have taught this enjoyable subject.

Comments and questions (and corrections!) on this script can always be addressed per E-Mail to the address: socolofs@ifh.uka.de.

Karlsruhe,
October 2002

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